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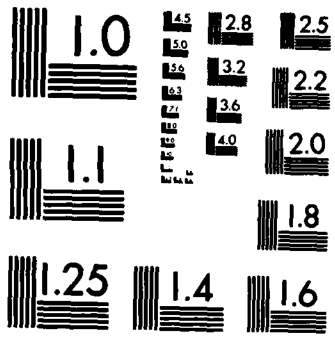
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# GAME-THEORETIC ANALYSES OF TRADING PROCESSES

by

Robert Wilson

Technical Report No. 474

August 1985

THE ECONOMICS SERIES

INSTITUTE FOR MATHEMATICAL STUDIES IN THE SOCIAL SCIENCES

FOURTH FLOOR, ENCINA HALL

STANFORD UNIVERSITY

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## ABSTRACT

Three topics are discussed. The first is a research program to establish whether the familiar trading rules, such as sealed-bid and oral double auctions, are incentive efficient over a wide class of economic environments. The second is a review of recent studies of dynamic trading processes, and particularly the effects of impatience and private information on the timing and terms of trade; the main emphasis is on models of bilateral bargaining. The third considers prospects for embedding bargaining and auction models in larger environments so as to endogenize traders' impatience as a consequence of competitive pressures; models of dispersed matching and bargaining and a model of oral bid-ask markets are mentioned.

GAME-THEORETIC ANALYSES OF TRADING PROCESSES*

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Robert Wilson

## Introduction

My aim in this paper is to describe some developments in the theory of exchange. The topics I describe share a common focus, namely the determination of the terms of trade. They also share a common methodology, which is the application of game theory to finely detailed models of trading processes. The aim of this work is to establish substantially complete analyses of markets taking account of agents' strategic behavior. Typically the results enable two key comparisons. One is the effect of altering the trading rules, and the other is the effect of alterations in the environment, such as changes in the number, endowments, preferences or information of the participants. Beyond these comparisons, however, the results are building blocks in the construction of a genuine theory of price formation. There are also important welfare consequences: the choice of trading rule determines the magnitude and distribution of gains from trade among the agents.

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I mention two caveats. One is that the many contributors to this work are not a team with a unified research program; rather, I perceive a shared belief that advances in game theory enable direct approaches to the problem of price formation. The other is that I address only a few topics in which I have been engaged recently, without any attempt to survey the field or mention all the relevant contributions. In particular, I confine attention to some exchange models with explicit trading rules. This excludes, for example, the field of industrial organization, which has enjoyed the most progress from application of game-theoretic methods; cf. Roberts [1985]. I regret omitting the work on markets mediated by specialists (e.g. Rubinstein and Wolinsky [1985]), particularly the contributions that examine the role of traders with inside information; cf. Glosten and Milgrom [1983], Hagerty [1985], and Kyle [1981]. And for some topics I do discuss, such as auctions and bargaining, I defer to other addresses and contributors at this congress for more complete treatments: Milgrom [1985] and Rubinstein [1985b].

The game-theoretic method is usually interpreted as employing models that specify explicitly the contingencies in which economic agents take actions. I agree but place the emphasis on the role of common knowledge. As Sergiu Hart has remarked, the common knowledge comprises the rules of the game. In practice, however, there may be little that is common knowledge. Game theory has a great advantage in explicitly analyzing the consequences of trading rules that presumably really are common knowledge; it is deficient to the extent it assumes other features to be common knowledge, such as one agent's probability assessment about another's preferences or information. I foresee the progress of game theory as depending on successive reductions in the base of common knowledge required to conduct useful analyses of practical problems. Only by repeated weakening of common knowledge assumptions will the theory approximate reality. But game theory as we presently know it cannot proceed without the fulcrum of common knowledge.

I have chosen three themes. The first is the matter of the efficiency of trading rules. I discuss the prospect that the familiar trading rules found in established markets can be verified to be incentive efficient in the sense of Holmström and Myerson [1983]. If successful, this effort would establish results comparable to those established for the Walrasian model, but in this case with explicit attention to strategic behavior and private information. The second theme is the role of time in exchange processes. Presently we know little about dynamics, but already the study of bargaining models has revealed that intertemporal features combined with asymmetries of information can greatly affect the terms of trade and that efficiency can be adversely affected by delay costs. On the other hand, if traders are patient or offers are rapid, these costs can be eliminated, but only by skewing the distribution of the gains from trade: this is the Coase [1972] conjecture that plays a central role in the theory. My final theme is a speculative essay on the prospects for synthesizing a theory of complex markets from simpler ingredients. I mention recent contributions that build models of exchange from particular models of bargaining and auctions.

Throughout I restrict attention to situations in which traders have inelastic demands for or supplies of a single unit of consumption at a valuation or reservation price that may be privately known. No risk aversion or wealth effects are included. Probability distributions are always taken to be common knowledge among the traders. By an equilibrium I shall always mean a sequential equilibrium, for which a subgame-perfect equilibrium suffices if there is complete information. Not all of the assumptions are specified for the models considered; hopefully they are clear from the context but in any case consult the references mentioned. The appendix includes several brief specifications of models mentioned in the text.

## 1. Incentive Efficiency of Trading Processes

By a trading rule I mean a specification of the actions available to the agents in each contingency, together with a function specifying the outcome (an allocation) resulting from each combination of the agents' actions. Given a trading rule, a strategy for an agent specifies which action to take in each contingency, depending for example on his preferences and other private and public information. Thus, each trading rule induces a game among the agents--a game of incomplete information (Harsanyi [1967]) if the agents have private information. To predict the outcome of this game, we rely on a selection of one of the sequential equilibria (Kreps and Wilson

[1982]). For welfare comparisons, say that one trading rule dominates another if all agents prefer the outcome of the first to the outcome of the second; and say that an undominated trading rule is efficient. When the agents have private information, this notion must be amplified (cf. Holmström and Myerson [1983] and Wilson [1978]): one rule dominates a second if it is common knowledge among the agents that all agents prefer the first to the second, as measured by their conditional expected utilities of outcomes. This definition yields the efficiency criterion called <u>interim</u> or incentive efficiency; the stronger criterion of <u>ex ante</u> efficiency evaluates the agents' preferences before they receive their private information, in terms of their unconditional expected utilities. In the use of this criterion it is important to identify the feasible set of trading rules: each outcome must be an allocation, and each agent must have an <u>interim</u> incentive to participate. In particular, this restriction excludes rules developed by Groves [1973] and d'Aspremont and Gerard-Varet [1979].

The study of efficient trading rules originated with Myerson's [1979] study of bargaining and his [1981] characterization of auctions that are optimal for the seller, and subsequent work has successfully characterized efficient trading rules in other environments.[1/] The key technique in this work has been the revelation principle; cf. Myerson [1979] among others. Essentially this uses the fact that to every trading rule and equilibrium, corresponds another trading rule with the same outcome, for which an equilibrium specifies only that the agents report truthfully their private information. The revised trading rule is simply the composition of the original trading rule and its equilibrium strategies. Thus, among the efficient trading rules is one inducing a 'direct revelation game'. The power of this approach is seen most clearly in the results of Gresik and Satterthwaite [1983] showing that it is possible, for a class of environments, to construct _ex ante_ efficient trading rules whose expected unrealized gains from trade decrease with the number of agents--in inverse proportion to the _square_ of the number of agents in the case of uniformly distributed valuations. Their results indicate that surprisingly few agents are required to obtain most of the gains from trade realized by 'perfect' competition or with complete information.

This brings me to a point I want to emphasize. The optimal trading rule for a direct revelation game is specialized to a particular environment. For example, the rule typically depends on the agents' probability assessments about each other's private information. Changing the environment requires changing the trading rule. If left in this form, therefore, the theory is mute on one of the most basic problems challenging the theory. I refer to the problem of explaining the prevalence of a few simple trading rules in most of the commerce conducted via organized exchanges. A short list, including auctions, double auctions, bid-ask markets, and specialist trading, accounts for most organized exchange. Indeed, bid-ask markets such as conducted in the commodities pits have long been economists' paradigms for the nearly-perfect markets addressed by the Walrasian theory of general equilibrium. The rules of these markets are not changed daily as the environment changes; rather they persist as stable, viable institutions. As a believer that practice advances before theory, and that the task of theory is to explain how it is that practitioners are (usually) right, I see a plausible conjecture: these institutions survive because they employ trading rules that are efficient for a wide class of environments. The experimental evidence, moreover, reinforces this view; cf. Plott [1982] and Smith [1982].

A useful next step in the study of trading processes is to verify the efficiency of the several familiar trading rules. This research program poses an analytical task that is the reverse of the approach derived from the revelation principle. Using the revelation principle, one can construct for each environment a direct revelation game that is efficient. Unfortunately the trading rule obtained this way depends on the common knowledge structure of the environment. In contrast, the familiar trading rules specify procedures that are independent of such data: typically they merely process bids and offers. The task is to show that some rule, or a specific candidate rule, of this special kind is efficient--and not just for one environment, but uniformly over a wide class of environments.

This task is actually a familiar one, at least in spirit. The analyses of the Walrasian model in the 1950's specified a particular trading rule and then showed that it yielded efficient outcomes for a wide class of environments (characterized mainly by convexity properties). We often ignore the trading rule underlying the Walrasian model, but it is clearly there: if the distribution of preferences and endowments is common knowledge, the market clearing price is computed and announced, and then agents receive their reported preferred net trades. Sonnenschein [1974] develops an axiomization of rules that rely on a public signal followed by private responses, and shows that such rules are essentially equivalent to a Walrasian price system.

I have attempted this task for the case of a double auction, in which buyers and sellers submit sealed bids and offers and a market clearing price is selected. I can report that it is quite complex; cf. Wilson [1985c]. No simple argument based on a separating hyperplane suffices as in the Walrasian model; apparently novel mathematical aspects are involved. The key tool available is Myerson's [1981, 1985] condition: an efficient rule must maximize the gains from trade that would result were the agents' valuations replaced by their virtual valuations, in which each agent's valuation is modified by a term reflecting incentive constraints. This term depends endogenously on the common knowledge structure of the trading game as well as the trader's valuation, so it is generally difficult to compute. Fortuitously, however, the trading rule for a double auction maximizes the gains from trade that would result were the agents' valuations replaced by their submitted bids and offers. Thus, it has a form quite similar to the one required for efficiency, except that the way in which imputed gains are measured differs; moreover, the maximization of these gains is an operation that depends only on ordinal properties, so any ordinally equivalent representation of the gains from trade yields the same allocation. The proof of efficiency reduces, therefore, to a demonstration that the agents' bids and offers are related by a monotone transformation to their virtual valuations. Some details are described in §A.1 of the appendix. In my work, I employed an ad hoc guess to construct such a transformation, and the conclusion was consequently rather weak: with various restrictive assumptions [e.g., independently and identically distributed valuations] a double auction is interim efficient if the numbers of buyers and sellers are sufficiently large. A corollary is that with symmetric equilibria all of the agent's welfare weights coverge to unity as the numbers of buyers and sellers increase, and the double auction trading rule is asymptotically ex ante efficient. A more precise examination of the conditions for existence of the requisite transformation might yield a stronger result. I know of no mathematical tools that address this problem directly.

What we know about auctions and double auctions suggests an interesting speculation. Each of these trading rules selects the allocation that would be efficient (maximizing the gains from trade) if the agents' submitted bids and offers were their true valuations, as would be the case, for example, were the numbers of buyers and/or sellers infinite. It would be 'nice' if fairly generally the rule that maximizes the apparent gains from trade as measured by submitted bids and offers is efficient. Such a rule is a natural candidate for a uniformly efficient rule, since it works when there is complete information and also when there is incomplete information if there are infinitely many agents. This hypothesis would say that with finite numbers the agents' equilibrium strategies for submitting bids and offers take account of the incentive constraints in precisely the way required to realize effiency. A counterexample would be equally interesting.

I summarize these remarks as follows. The program that demonstrated the existence of equilibrium for the Walrasian model and established the efficiency of the resulting allocation for a large class of environments, has a current counterpart in a program to establish the existence of equilibrium and the uniform efficiency of the trading rule for some of the familiar market mechanisms that include explicit procedures for price formation based on agents' submitted bids and offers. The success of such a program would establish a cornerstone for economic theory; its failure, say by significant counterexamples, would raise challenges to both theory and practice. As I suggest below, extension of this program to dynamic trading processes will be important; an interesting recent contribution by Gale [1984a, 1984b, 1985] will be discussed later.

## 2. Time and Impatience in Dynamic Trading Processes

The theory of efficient trading processes developed in a static framework relies heavily on the assumption that repetitions are precluded. For example, the design of an auction that is optimal for the seller, as derived in Myerson [1981], includes an optimal reservation price that exceeds both the seller's valuation and the buyers' least possible valuation. Thus there is a chance that the buyers' valuations are insufficient to elicit acceptable bids, yet gains from trade are present. In this case, if no acceptable bid is received, then the seller and the buyers share an incentive to reopen the bidding with a lower reservation price.[2] Another way to see this is to consider a Dutch auction in which the seller reduces the asking price until some buyer accepts or the seller terminates the auction. In any sequential equilibrium of the game, the seller continues to reduce his asking price so long as a chance remains of gains from trade. Similarly, in the double auctions studied by Myerson and Satterthwaite [1983], Chatterjee and W. Samuelson [1983], and Wilson [1985c, 1986a], there is a chance that not all the gains from trade are realized, and consequently there is an incentive to reopen trading. Cramton [1984a] has emphasized, therefore, the importance of studying the so-called 'perfect' market games that allow continuation (e.g., repetition) so long as gains from trade remain likely. Models that allow such continuation have the advantage of avoiding *a priori* presumptions that commitments to terminate trading are credible. Assured continuation is a significant restriction (e.g., a seller cannot make a final 'take it or leave it' offer) but it gains realism. It also has important distributional consequences, since the gains from trade are usually allocated quite differently and there may be substantial costs of delay incurred; on the other hand, all gains from trade are eventually realized.

In this section I offer remarks about several recent studies of market games allowing endless continuation. Mainly I comment on the role of time in trading processes and the important effects of agents' impatience to conclude trades. I divide the discussion between market games with and without complete information.

## 2.1 Dyanamic Market Games with Complete Information

The key contribution is Rubinstein's [1982] study of a bargaining game in which a buyer and a seller of an indivisible item alternate bids and offers until one accepts. Rubinstein shows that, for a restricted class of preferences exhibiting stationarity and impatience, this game has a unique subgame-perfect equilibrium. Trade occurs immediately at a price that divides the gains from trade according to the parties' relative impatience, say as measured by their discount factors as I will assume hereafter.3/

Rubinstein's formulation allows many extensions, of which I mention one (Wilson [1985d]) with many buyers and sellers, each with one unit to buy or sell. Suppose that the sellers simultaneously each offer an ask price; then buyers simultaneously respond, with each either accepting any seller's offer or making a counteroffer of a bid price; *et cetera* with the buyers and sellers alternating roles. Assume that tied acceptances are resolved to maximize the realized gains from trade. Then again there is a subgame-perfect equilibrium in which the outcome is efficient: all trade occurs immediately and gains from trade are exhausted. Also, all accepted prices are the same, and this price is a Walrasian market clearing price.4/

Variants of Rubinstein's model have been useful in studying other market structures. Notable instances in industrial organization theory are the models analyzed by Maskin and Tirole [1982 *et seq.*] in which firms alternate in making two-period commitments to their production plans.

In Gul, Sonnenschein, and Wilson [1985] we study the case of a single seller and a continuum of buyers. Assume that the seller has a constant unit cost of supply and that the distribution of the buyers' valuations (each for a single unit of consumption) is known to the seller; also, all the buyers (and for simplicity the seller) have the same discount rate. In a fashion similar to Rubinstein's bargaining model, allow that the seller offers a price each period, which each buyer can accept or reject, but in this model exclude counteroffers by the buyers. Again, focus attention on the subgame-perfect equilibria of this game. This formulation provides a basic model of monopoly when the seller cannot restrain his output rate nor commit to a particular path of prices. Some technical aspects of the formulation are described in §A.2.

The analysis of this game divides into two cases depending on whether the seller's cost is less than all the buyers' valuations. If it is, then when sufficiently few buyers remain, the seller offers the maximum price that will clear the market. This feature allows a construction of the equilibrium by backward induction much as in dynamic programming. As shown by Fudenberg, Levine, and Tirole [1983] and in Gul, Sonnenschein, and Wilson [1985], the market is cleared after a finite number of offers from the seller, and there is a unique subgame-perfect equilibrium. A novel feature is that the buyers expect the seller to use a non-stationary randomized strategy off the equilibrum path. However, an important simplifying feature is that the buyers have strategies that are pure and stationary: each buyer's strategy specifies a reservation price and he waits to accept until that price or one lower is offered. Also, the seller makes only 'serious' offers: each is accepted by some buyers.

If the seller's cost is not less than all the buyers' valuations then the matter is more complicated. Even for the simple case that the buyers' valuations are uniformly distributed (i.e., a linear demand function), we exhibit a continuum of equilibria in stationary strategies; moreover, these can be pieced together to generate equilibria in non-stationary strategies. All of these equilibria have different price paths. Thus in this case the game-theoretic analysis reveals much more indeterminancy and complexity than is often ascribed to monopoly behavior. The source of this phenomenon is that there are many price paths that can be anticipated by buyers and that are optimal for the seller, since the infinite continuation of the game precludes pinning down a unique equilibrum by working backward from the terminus. Even with complete information, indeterminancy of 'rational expectations' is possible.

A major result in Gul, Sonnenschein, and Wilson [1985] is a general verification, for the case that buyers' strategies are stationary, of a conjecture due to Coase [1972]. (Here, stationarity means that each buyers' maximal acceptance price is independent of the seller's previous history of offers.) As the duration between the seller's offers shrinks to zero along the equilibrium path, the seller's ask prices converge to the maximum of his cost and the minimum among the buyers' valuations. That is, with frequent offers, the outcome is approximately Walrasian: all trades occur early at prices near the maximum Walrasian market-clearing price. As Coase conjectured, when a monopolist can neither commit to future prices nor limit his production rate, his market power is severely eroded if either buyers are patient or the rate of offers is high.

It is not easy to develop an intuitive appreciation of this result, but here is a try. The key consideration is that, since the buyers' strategies are stationary, the seller has the option at any time to accelerate the process by offering tomorrow's price today and thereby advancing the acceptance dates of subsequent buyers. The cost of doing this is the foregone higher profit on those buyers accepting today, whereas the benefit is the interest on the seller's present value of continuation, which is thereby made to arrive a day earlier. Since an equilibrium requires that exercising this option must be disadvantageous for the seller, we know that the cost must exceed the benefit. But the cost is approximately the price cut times the number of buyers who accept today's price, and the benefit is the daily interest on the continuation value. Consequently, the daily interest on the continuation value is bounded by approximately the day-to-day price drop times the number accepting per day. Fix the interest rate per unit time to be 100%, and divide this inequality through twice by the length of a day: then the continuation value divided by the length of a day is bounded by the product of the rates (per unit time) at which prices decline and buyers accept. As the length of a day shrinks, the rate of price decline must be bounded or buyers would prefer to wait rather than accept the current price. If the rate of acceptance is also bounded, then as the length of a day shrinks the continuation value must also shrink to zero--if opportunities remain for the seller to reduce his price. If the continuation value shrinks to zero then the seller's later prices must all be converging to his unit cost, and therefore his present prices too: otherwise, if the day is sufficiently short then the buyers all prefer to delay purchasing. If no opportunities for further price reductions remain, then the price must already be at its minimum, which is the minimal valuation among the buyers. The remaining case, therefore, is that the rate of acceptances is unbounded. But in this case also the prices offered by the seller must all be converging downward to his unit cost (or the buyers' least valuation), since this is the only way that a positive fraction of the buyers will accept in each of several days when their interest cost of delay is small; that is, the sequence of prices must become flat in the limit, yet the sequence is tied down at the end. In outline, this is one interpretation of the arguments supporting the Coase conjecture.[5] The complete proof is much more complicated, of course.

The exploration of these ideas has been a central topic in the literature on the durable-good monopoly problem, which is essentially equivalent to the one posed above; cf. Bulow [1982] and Stokey [1982]. They are also discussed briefly in Maskin and Newbery [1978]. As we shall discuss later, the Coase conjecture also has important ramifications for the study of bargaining with incomplete information.

## 2.2 Dynamic Market Games with Incomplete Information

Extensions of Rubinstein's bargaining model to situations with private information is currently the most active research topic in this area. Here I mention briefly a few recent results that raise issues of general interest. For simplicity, assume that the seller and the buyer have the same discount rate. Also, assume throughout that the buyer's valuation is privately known, and distributed independently of the seller's according to a distribution that is common knowledge. As in Cramton [1984a, 1984b], say that an offer is serious if it has a positive probability of being accepted.

Among the possible trading rules with possibly endless continuation, three of interest are the following:

(S) Only the seller makes offers and the buyer merely waits to accept some ask price:

(B) Only the buyer makes offers and the seller waits to accept some bid price; and

(A) The seller and the buyer alternate making offers until one accepts the other's offer.

### 2.2.1 Bargaining with Private Information on One Side

First consider the case that the seller's valuation is common knowledge and the buyer's valuation is privately known.

Trading rule (S) in which only the seller makes offers has been studied by Sobel and Takahashi [1983], Fudenberg, Levine, and Tirole

[1983], Cramton [1984a, 1984b] and Gul, Sonnenschein, and Wilson [1985]. The key observation is that the characterization of the sequential equilbria of this game and the characterization of the subgame-perfect equilibria of the monopoly game are formally equivalent (see §A.2). That is, the situation of a seller repeatedly making offers to sell a single item to a single buyer with privately known valuation is equivalent to the situation of a seller repeatedly making offers to sell many units to a population of many buyers with valuations known to be distributed according to the same distribution function for the privately known valuation of the buyer in the bargaining situation. Thus, all of the results described above for the monopoly game apply to this bargaining game, including for example the verification of the Coase conjecture for equilibria with stationary strategies for the buyer. Also, if the buyer's valuation surely exceeds the seller's then there is a unique equilibrium obtained by backward induction from the final offer of the seller that the buyer is sure to accept.

Trading rule (B), in which only the buyer makes offers, presents a game with significantly different features, because the buyer's offers potentially reveal his valuation. I don't recall an exposition in the literature but the main ideas are implicit in Cramton [1984a, 1984b]. There are many sequential equilibria since the variety of responses by the seller to disequilibrium offers by the buyer can support an equal variety of equilibrium signaling strategies by the buyer. One salient class of equilibria comprises those in which the buyer signals his valuation by his willingness to delay making a serious offer. Typically these require nonstationary strategies for both parties, as a way of coping with the buyer's incentive to defect from the seller's prediction of his behavior. See §A.4 for more details. When offers are made continuously, an extreme case is the trivial equilibrium in which the buyer offers only the price zero and the seller accepts any price that is at least zero. This is apparently the only equilibrium in stationary strategies: given stationarity, it is always in the interest of the buyer to accelerate the process so as to avoid interest costs. In view of the result obtained for trading rule (S), that with stationarity and frequent offers the informed party captures all the gains from trade, this equilibrium has special significance. It shows that the same result can obtain if the informed party makes all the offers.

Trading rule (A) in which the seller and the buyer alternate offers has been studied by Grossman and Perry [1984] and Gul and Sonnenschein [1985], among others. The principal result is a theorem of Gul and Sonnenschein about equilibria in which the buyer's strategy is stationary.6/ Informally, their theorem states that the probability that a trade is not concluded within any initial time interval can be made arbitrarily small by making the period between offers sufficiently short. To see the ramifications of this theorem, consider the case that the least possible valuation $v_*$ of the buyer exceeds the seller's valuation of zero. Let $\Delta$ be the period length and let $n(\Delta)$ be the maximum number of periods that can transpire before trade is concluded along the equilibrium path: then the theorem states that $\Delta n(\Delta) \to 0$ as $\Delta \to 0$. Thus, the time required to complete the transaction is small if the period length is short, and in the limit trade occurs immediately; moreover, the limit price is necessarily no more than $v_*$. We see again that the informed party obtains most of the gains from trade if the period length is short--another version of the Coase conjecture.

In Wilson [1985a] I develop some further consequences of this striking result for equilibria of the form studied by Grossman and Perry. In their construction a serious counteroffer by the buyer is necessarily the Rubinstein offer for the least among the buyer types making that offer in equilibrium. That is, along the equilibrium path, if a serious counteroffer is made by buyer types with valuations in the interval $[x,y]$, after a history that enables the seller to restrict the support to $[v_*,y]$, then this offer is $p^0(x) \equiv \delta x/(1+\delta)$ where $\delta = e^{-r\Delta}$ is the discount factor. For equilibria of this type, the theorem of Gul and Sonnenschein implies that for any particular buyer valuation $v$, if the period length is sufficiently short then the buyer makes no serious counteroffers except possibly the minimal one $p^0(v_*)$. This suffices to explain why Grossman and Perry find that an equilibrium of the kind they specify can exist only if the discount factor is sufficiently small.7/

To illustrate these features, consider the example in which the buyer's valuation is uniformly distributed between zero and one. In this example the Grossman-Perry equilibrium does not exist if $\delta > .8393$. More interesting however is the way in which their equilibrium behaves as the discount factor increases towards this critical level: the length of the interval of buyer types making a particular counteroffer shrinks to zero. At this critical level of the discount factor, the equilibrium changes continously to one in which the buyer of any type always counteroffers with the nonserious offer of zero. Thus, for larger discount factors only the seller makes serious offers and the buyer waits to accept a price that is sufficiently low considering this valuation. In such a game (Gul, Sonnenschein, and Wilson [1985]), we know that the Coase conjecture is satisfied for any equilibrium with a stationary strategy for the buyer. This seems to be a main explanation for the Gul-Sonnenschein theorem. See §A.3 for more details.

The main parameters of the equilibria for this example are tabulated in Table 1, using the following notation. When the support of the buyer's valuation is $[0,\mathbf{x})$ the seller's offer is $\mathbf{p(x) = Ax}$, which is accepted by the buyer if his valuation is in the interval $[\boldsymbol{\alpha}\mathbf{x},\mathbf{x})$ and rejected otherwise, whereupon the buyer makes the serious counteroffer $\mathbf{p}^0(\boldsymbol{\alpha\beta}\mathbf{x})$ if his valuation is in the interval $[\boldsymbol{\alpha\beta}\mathbf{x},\boldsymbol{\alpha}\mathbf{x})$ and this counteroffer is accepted by the seller. Note that a counteroffer is possible only if $\boldsymbol{\beta} < 1$, which occurs only if $\delta < .8392867552$. From the support $[0,\mathbf{x})$ the seller's expected present value of continuation is $\frac{1}{2}\mathbf{Ax}$; i.e., $2\mathbf{A}$ is the fraction of the expected profit the seller could obtain were he able to make a final 'take it or leave it' offer. We omit here the description of the off-the-equilibrium-path behavior except to note that in one version the seller employs a randomized acceptance strategy in response to a counteroffer in an interval below the one expected in equilibrium; Grossman and Perry [1984] suggest another version.

Table 1

| $\delta$ | $\alpha$ | $\beta$ | A |
|---|---|---|---|
| .80 | .692 | .679 | .306 |
| .83 | .668 | .908 | .342 |
| .8392867552 | .6477988713 | 1.000 | .3522011287 |
| .84 | .64826 | | .35174 |
| .90 | .69643 | | .30357 |
| .95 | .76205 | | .23795 |
| .99 | .87637 | | .12363 |

There are several main conclusions to be drawn from the known results about trading rules (S), (B), and (A) when the seller's valuation is common knowledge. One is surely the strong confirmation of the Coase conjecture when the seller makes offers and the buyer has a stationary strategy. The seller's bargaining power is severely eroded if the buyer has private information, has the option to pass, and is patient--at least if the buyer's strategy is stationary. This prediction from the game-theoretic analysis has important practical applications, and it is a prediction that is suitable for experimental testing. On the other hand, the key role of the stationarity of the buyer's strategy suggests caution. Stationarity is necessary only for trading rule (S) and only for the case that the existence of positive gains from trade is common knowledge. We know little about the equilibria with non-stationary strategies, yet as we shall see later they can play an important role in bargaining with uncertainty in determining the existence of gains from trade.

Trading rule (A) has also been studied in a different context by Rubinstein [1985a] and by Bikhchandani [1985]. These authors consider the case that the gains from trade are common knowledge, say the seller's valuation is zero and the buyer's is one, but the seller is uncertain about the buyer's discount factor. Both study a model in which it is common knowledge that the seller's discount factor is $\delta$ and the buyer's is either $\delta_s$ or $\delta_w < \delta_s$ with specified probabilities that are common knowledge; of course the buyer knows his discount factor. Noting that this game has many sequential equilibria, Rubinstein imposes a set of conditions that are sufficient to identify a unique equilibrium in which, in the interesting case, the seller makes an initial offer that the impatient buyer accepts but the patient buyer rejects and counteroffers with a bid that the seller accepts. Since these results are described by Rubinstein [1985b] in his address at this Congress, I will not explore his construction further here except to endorse Bikhchandani's observation that a key feature is disequilibrium behavior that is possibly worrisome: the probability that the seller assigns to the prospect that the buyer is impatient after seeing a counteroffer is neither continuous nor monotone in this counteroffer. In particular, the expected counteroffer is conclusive evidence that the buyer is patient (by Bayes' rule), whereas a slightly smaller counteroffer leaves the seller's prior assessment unchanged. To alleviate this difficulty, Bikhchandani constructs an alternative equilibrium that can be described briefly as follows for the case that $\delta_s^2 < \delta_w$.[8/] Each time it is the buyer's turn, he offers the seller his expected value of continuation, to which the seller responds with a randomized acceptance rule. At his turn, the seller similarly offers a price that the impatient buyer is indifferent about accepting, and he too responds with a randomized acceptance rule (the patient buyer surely rejects). Both the buyer's and the seller's prices decline over time, but of course at any time, the seller asks more than the buyer bids. As the process continues without an acceptance, the seller's probability assessment that the buyer is impatient becomes increasingly pessimistic until after a finite number of periods, the seller asks or surely accepts the Rubinstein offer were the buyer known to be patient.[9/] This equilibrium is qualitatively different than Rubinstein's in that the bargaining can extend over numerous periods and the gains from trade can be split in numerous ways--mainly because of the randomized acceptance rules used by both the seller and the impatient buyer.

I see the two dramatically different equilibria proposed by Rubinstein and Bikhchandani as an interesting test case for experimental studies. A persistent difficulty in the study of bargaining is the plethora of equilibria when there is incomplete information. Here we have a model simple enough for an experimental design and two quite different equilibria with plausible merits that appear susceptible to definitive empirical confirmation or rejection. As we make headway in choosing among these two equilibria and others, we will learn better how to select among plausible equilibria in more complicated problems.

### 2.2.2 Bargaining with Private Information on Both Sides

I turn now to bargaining games in which both parties have private information. Any approach to this subject must contend with the ramifications of the Coase conjecture that has been established for the case that only one party has private information. Essentially this says that if the period between offers is short, then in any reasonable equilibrium with a stationary strategy for the informed party, the other party with inferior information captures little of the gains from trade--essentially because of the incentive to accelerate the process. In bargaining with both sides having private information, therefore, if periods are short and the equilibrium is stationary, then each party is deterred from making an offer that might reveal his valuation--doing so would substantially eliminate his gains from trade. Thus, the dilemma has two horns: one can either have a separating equilibrium using non-stationary strategies to avoid the Coase conjecture, or some kind of pooling equilibrium using stationary strategies. So far, the latter approach has not been pursued except in a version with strategies restricted to stopping times by Chatterjee and L. Samuelson [1984], but suggestions are included in Gul and Sonnenschein [1985]. Cramton [1984a] has developed the former approach using strategies in which delay in making or accepting a serious offer is a trader's means of credibly signaling his valuation. In the equilibria he constructs, the nonstationarity is localized in the process by which beliefs are revised off the equilibrium path.

First consider trading rule (S) in which only the seller makes an offer in each period. Assume for simplicity that both parties have the same discount factor and that the seller's and buyer's privately known valuations are uniformly distributed on the same interval. Although Fudenberg and Tirole [1983] and Cramton [1984a] establish in two-period models that a partially pooling equilibrium can be advantageous for the seller, in the infinite-period model discussed here, a separating equilibrium is analyzed. In such an equilibrium along the equilibrium path, the seller initially delays making a serious offer in order to signal that his cost is not very low, until he makes a serious offer than enables the buyer to infer his cost precisely. This first serious offer has positive probability of being accepted; the buyer accepts if his valuation is sufficiently high. If this first serious offer is not accepted, then a second phase ensues in which the seller continues with successively lower offers (declining to his cost) as in a Dutch auction until the buyer accepts (if there are gains from trade) at a price depending monotonically on his valuation and the discount rate. In this second phase, the seller is deterred from accelerating the process by nonstationary responses of the buyer: unexpectedly low offers are interpreted as convincing evidence that the seller's cost is lower than originally inferred, and if the seller's inferred cost is zero then the buyer expects to obtain a price of zero and thereafter insists on it. Given this anticipation of the second phase, the seller's strategy in the first phase is supported by two considerations. First, assuming that all nonserious offers are equally uninformative to the buyer, the seller can make an offer sufficiently above the buyer's acceptable level in order to effect the delay that signals that his cost is not very low. Second, contemplating a serious offer, the seller trades off two considerations. Making an immediate serious offer brings a chance that it will be accepted, or in any case obtains the value of continuation in the Dutch auction based on his true cost while following the strategy were his cost the lower one inferred by the buyer depending on which of the possible serious offers the seller makes, hence the seller assesses an interest cost on this foregone continuation value that is immediately accessible. On the other hand, delaying another period signals that his cost is higher and induces a higher continuation value in the second phase. Balancing these two considerations and choosing an optimal offer when the time comes, with both the time and the offer depending on his cost, the seller eventually makes a serious offer that reveals his cost precisely.

The feature of such a construction that makes the equilibrium 'work' is the seller's signaling motive: at any time a serious offer is higher than the price he would offer were his cost common knowledge, since it must be high enough to be credible by assuring that if his cost were lower, then he would have no incentive to imitate such an offer. Of course this signaling is expensive for the seller: it reduces the chance the buyer will accept. As a result of this signaling incentive, serious offers are a sharply increasing function of the seller's cost, and the range of prices at any time has an upper bound beyond which there is no chance the buyer will accept. The buyer can do better than accepting an exorbitant (i.e., nonserious) offer by waiting one period for a lower offer from the seller, since in the next period the seller, having a diminished signaling motive (since a range of lower possible costs is now excluded from the buyer's assessment of the seller's cost), can be expected to make a serious offer less inflated by the pressure to signal credibly. This bound establishes the range of serious offers and thus assures the existence of nonserious offers that the seller can make to effect delay. The net result is that along the equilibrium path, at successively later times successively higher intervals of costs induce the seller to make a serious offer. These serious offers are increasing in the seller's cost and therefore are revealing, but they are not monotone over time since for each particular cost, the pressure to signal credibly diminishes with time as the buyer truncates from below the support of his assessment as the history of nonserious offers is extended.

The plausibiltiy of this equilibrium clearly depends on the central issue of whether the second phase's reliance on nonstationary responses by the buyer off the equilibrium path--and thereby the circumvention of the Coase conjecture's implication when the discount factor is large--can reflect actual behavior. If the second phase is accepted then the seller's incentive to signal by delay and to make a revealing offer follows convincingly. If stationary behavior by the buyer is assumed, on the other hand, then with short periods, the seller is deterred from making a revealing offer by the anticipation that there will be little gain from trade realized in the second phase. There is, however, the possibility that partially revealing offers by the seller, also studied by Cramton [1984a], should come into play in this case: for the two-period examples Cramton studied these yielded very small benefits to the seller, but in many-period models with short periods one can expect that these benefits will loom large.

Cramton has extended his construction to trading rule (A), involving alternating offers, by employing a continuous-time model interpreted as the limit of such a process. This interpretation is invoked by assuming that if at any time both parties' valuations stand revealed (by inference from serious offers) then gains from trade are split immediately in proportions specified by Rubinstein's model with alternating offers and complete information. From this assumption, the construction works backward much as before. If one trader's valuation is revealed, say the seller, then he conducts a Dutch auction, now in continuous time with offers declining continuously to his revealed cost. The novel feature is that in the initial phase both parties use delay followed by a serious offer to signal their valuations.[10] Of course in considering a first serious offer a trader now sees a further advantage to waiting: there is a chance that the other will make a serious offer first, which is advantageous to the one who waits. The equilibrium conditions derived from this construction yield differential equations that in special cases can be solved to obtain the traders' strategies explicitly. I defer elaboration of Cramton's construction until Section 3 where I describe some of its implications in the context of bid-ask markets.

An equilibrium of this form poses clearly the dilemma in the second phase between nonstationarity and stationarity. Using a nonstationary construction for the off-the-equilibrium-path behavior, Cramton's equilibrium has the unrevealed trader making no serious offers until he accepts or offers the Rubinstein offer, which is itself strongly dependent on the alternation of serious offers. In contrast, with stationarity, the Gul-Sonnenschein theorem precludes the revealed trader from obtaining any of the gains from trade, and in particular for the Grossman-Perry equilibrium we have seen in the example reported in Table 1 that if the period length is short (zero for a continuous-time model) then the unrevealed trader makes no serious offers. My conclusion is that nonstationary equilibria fare poorly in this comparison; the best hope is that stationary equilibria with partial pooling can be developed to provide more plausible insights into the bargaining process, and I endorse the suggestions along this line by Gul and Sonnenschein [1985]. In the meantime, it remains true that Cramton's two equilibria for the trading rules (S) and (A) remain the only ones known (other than trivial equilibria enforced by extremely optimistic beliefs off the equilibrium path) and therefore provide our only developed analyses of bargaining with both parties having private information.[11] The search for stationary equilibria appears formidable and it may take considerable effort to explore fully its many complexities.

In summary of this section, I offer the following conclusions. It is abundantly clear that dynamic market processes, even the simplest cases of monopoly and bargaining, reveal awesome complexities and startling results. 'Rational expectations' is found to pervade even monopoly with complete information. With incomplete information, central to what we know so far are the myriad possibilities for signaling, delay, and the like. In dynamic processes, asymmetries of information interact strongly with traders' impatience. Most extraordinary is the Coase conjecture: asymmetries of information in trading processes that proceed rapidly can skew the terms of trade completely in favor of the informed party. Since this result depends heavily on stationarity, it calls attention to the critical behavioral role of stationarity and it throws into perspective the role of non-stationarity assumptions so often invoked in earlier models.

## 3. Synthesizing Theories of Markets

Much of economics concerns the theory of markets. Often we assess the contributions of this theory in terms of the analyses of models that it offers. Equally important, I think, are the models themselves. Models are designed to capture succinctly various features of practical situations. The formulation of models is an art form requiring insight and skill, as well as a sensitive appreciation of the features amenable to tractable analysis. Translating aspects of actual markets into consistent mathematical representations demands both keen perceptions of reality and mastery of the craft of construction.

There is also a meta-level of model formulation that deserves attention. At this level, the task is to assemble several models into a unified structure that describes a wider variety of economic environments according to a consistent scheme. Sometimes unification is achieved by generalization. The prominent example is the Walrasian model, which has been vastly generalized. I doubt that the game-theoretic analysis of markets will take this form. Presently the special structure of particular models must be exploited to obtain significant results; indeed, the comparative advantage of game theory is its elaboration of the fine structure of equilibria. An alternate path, however, relates to modeling the way that urban design relates to architecture. Progress on this path requires assembling large structures from smaller ones whose fine structure is modeled in detail.

There are now models describing market structures along the spectrum that ranges from 'perfect' competition with many agents down to bargaining between two agents. Along the spectrum are special cases such as (omitting the full list):

- Auctions: a single seller offering a single item to several buyers

- Double Auctions: many traders, each demanding or supplying a single unit.

- Monopoly: a single seller offering many units to several buyers.

The models formulated to examine these markets emphasize the structure implied by their particular features, expressed mainly in terms of the numbers, preferences and endowments, and/or information of the agents. A theory of markets gains explanatory power if these ad hoc formulations are knit together consistently, so that a single construct can be specialized to address any particular market structure along the spectrum. A truly successful endeavor would have comparable explanatory power in elucidating both bargaining and perfectly competitive markets, and would provide basic principles revealing how competition becomes perfect as the number and attributes of the participants changes.

### 3.1 Bargaining Models of Decentralized Exchange

An interesting contribution of this kind is the construction by Rubinstein and Wolinsky [1984] and Wolinsky [1985]. These papers build on the bargaining model of Rubinstein [1982] based on alternating offers and complete information, and imbed it in the fabric of a larger economy with many traders. To induce the feature of impatience so essential to the bargaining model, without imposing it exogenously, these authors develop a version of the 'competitive pressure' hypothesis (more on this below) by imposing on each bargainer a risk that his partner will find comparable (or in Wolinsky [1985], better but costly) bargaining opportunities elsewhere. Thus in equilibrium each pair trades immediately at a price determined by the relative risk each has of losing his partner--much as in the basic Rubinstein model, but with the novel feature that the gains from trade are computed relative to the alternative continuation values traders could achieve if their bargaining sequence were to be interrupted.

In Rubinstein and Wolinsky [1984], the authors envision a market with dispersed (identical) buyers and (identical) sellers who randomly encounter each other and then bargain together over the price via alternating offers. The stochastic assignment process that matches the traders for bargaining is exogeneous and stationary. Most important, having found a partner for bargaining, each trader runs a continuing risk that his partner may find another. If his partner leaves in some period, then the trader is left without a bargaining relationship for at least that period, after which he again stands a chance of finding a new partner. Thus failure to trade immediately imposes both the usual interest cost <u>and</u> the risk of incurring a further delay if the partner departs next period--this delay is the expected time until a new partner is found. The gist of this model, therefore, is that to the traders' direct impatience, represented by the interest rate, is added an additional term reflecting the 'competitive pressure' that one's bargaining partner may find another and leave one in the lurch until a new partner can be found. In equilibrium, of course, each pair trades immediately, at a price that reflects the relationship between the partners' <u>total</u> impatience as measured by the sum of these two terms.

As the interest rate or the length of a period shrinks to zero the predicted price at which each pair trades converges to a price that is not a Walrasian price for the economy as a whole, namely a price that equates the totality of demand and supply. This result differs from the corresponding result for the basic Rubinstein model, which yields a Walrasian price. The reasons for this deviation from the usual Walrasian prediction are explored in Binmore and Herrero [1984, 1985]. The essential technical feature is the fact that in the bargaining relationship, the gains from trade are computed relative to the traders' continuation values: if there are more sellers than buyers then the buyers have a proportionately greater chance of successfully finding another partner while the sellers have a proportinately greater chance of being left without a partner for a while. The end result is that the price at which they trade lies between their two valuations, in porportions that reflect these relative chances. In contrast, the only Walrasian price is equal to the sellers' valuation, which can result from a Rubinstein model only if the buyers' impatience is infinitesimal compared to the sellers'. Here their relative impatience is bounded, so the equilibrium price is not Walrasian.

As Binmore and Herrero [1984, 1985] emphasize, this feature arises from the assumed stationarity of the process. To maintain stationarity it must be that any initial disparity among the numbers of buyers and sellers must be sustained by arrivals of new traders in numbers equal to those departing with completed trades; hence, the surplus of traders on one side of the market is maintained at every time and there is no possibility that the market can actually clear in the Walrasian sense. Gale [1984a, 1984b, 1985] also concludes that stationarity is the essential ingredient. Both authors exhibit nonstationary processes that lead to Walrasian outcomes; one such model has a fixed population of traders.[12/]

I view these results as possibly realistic for the case of trade among dispersed, impatient agents who direct their offers to particular partners. For such cases, Rubinstein and Wolinsky effectively demonstrate that the continuing existence of asymmetries in the pool of unmatched traders can affect the terms of trade to the extent of yielding non-Walrasian outcomes if market clearing is perpetually delayed. Their model reveals the underlying assumption of the Walrasian model, that eventually the market can clear, usually by the imposed structure that eventually all traders get to take equal advantage of the trading opportunities. On the other hand, the analyses of Binmore and Herrero, and Gale, suggest that with market clearing and nil delay costs, Walrasian outcomes are to be expected.

Based on a seemingly comparable, though much generalized, model, Gale [1984b] argues that any equilibrium outcome must be Walrasian if markets clear and delay costs are nil. His formulation differs from the others we have discussed in that preferences are such that a Walrasian equilibrium is characterized by equality of the trader's marginal rates of substitution (thus the Rubinstein-Wolinsky model is excluded, but Gale [1985] follows Binmore and Herrero in arguing that this is inessential). Gale assumes outright that the interest rate is zero. In this case, he argues that any trader will continue trading (offering and accepting net trades according to a trading rule in which one trader is randomly selected to make a single 'take it or leave it' offer) so long as his marginal rate of substitution differs from the Walrasian price, since by continuing he obtains a positive chance of encountering another with whom an advantageous trade can be made. Thus, if the process stops at all, it stops at a Walrasian outcome. Assuming that endless continuation yields no consumption value, and that the stochastic matching process by which traders encounter each other is sufficient to ensure that the process stops after some finite time, Gale therefore argues that equilibrium outcomes are Walrasian.[13]

Unlike the previous models, Gale's model does not elucidate in detail the role of impatience, and in particular the role of competitive pressure is diminished to a search for the partners (several, if multilateral trades are required) who will provide the anticipated Walrasian net trades--who are assumed to be found eventually with no real delay cost. Strategic behavior is reduced to the mutual anticipation of each that only the predicted Walrasian net trade is acceptable, and each partner is willing to wait indefinitely to obtain that trade or to wait for a comparably good opportunity from a substitute. Though Gale's formulation seems somewhat displaced from the spirit of the other game-theoretic treatments, I find it particularly interesting as a fundamentally new motivation for the standard Walrasian model of exchange. Absent impatience, the Walrasian model can be justified as the net result of bilaterial bargaining among dispersed agents who encounter each other randomly, but with full knowledge of the terms of trade they can demand and should accept. As we have seen with the Rubinstein-Wolinsky model, this need not be the case if balanced departures and arrivals in the market prevent the market from ever clearing, and therefore there is no Walrasian clearing price that identifies unique marginal rates of substitution that can guide the agents in their searches for trades. But in other cases Gale's model, and the related ones by Binmore and Herrero, are likely to set a standard against which to compare the limits of other models as the interest rate or period length shrinks to zero.

### 3.2 Competitive Pressure in Auctions and Bid-Ask Markets

The principle that 'competitive pressure' induces impatience that affects the terms of trade can be further illustrated with the simple example of a Dutch auction conducted in real time. Suppose that there is a single seller and several potential buyers for an item. The seller's value is zero, whereas each buyer's valuation is privately known, but distributed according to a probability distribution that is common knowledge. In a Dutch auction conducted over a fixed time interval of duration 1, the seller starts with an asking price possibly as high as the highest possible valuation among the buyers, and then the asking price is reduced continuously to reach zero at time 1 (stopping at a price above zero can not be a sequential equilibrium strategy for the seller). The first buyer to announce acceptance of the current asking price receives the item by paying the price. In such an auction, equilibrium strategies for the buyers specify for each buyer, depending on his privately known valuation, the highest asking price (or the earliest time) that he will accept. The optimization of this acceptance price by a buyer involves the following tradeoff. At any instant, waiting a little longer reduces the price paid but incurs two charges against his present profit (the difference between his value and the price): one charge is the interest on the delay in receiving the profit, and the other is the hazard rate of the risk that some other buyer will intervene and accept the asking price first. Thus, the interest rate and the hazard rate of a competitor intervening occur additively in determining the optimal strategy. The interesting consequence of this observation is that even if the interest rate is zero, so that seemingly all buyers are patient enough to wait for the final price of zero, in fact the competitive pressure induces impatient behavior endogenously. As a result a buyer with a high valuation accepts early. The case that the interest rate is zero, at least for the seller, is particularly interesting because it enables the seller to be unconcerned about delay costs in conducting a Dutch auction; if his interest rate were positive and the buyers' strategies were stationary, then the seller would have an incentive to accelerate the process. For more details see §A.5.

The principle that competitive pressure induces impatience is, I think, an important explanation of behavior in more complicated real-time markets with many buyers and sellers. The prominent examples are markets with orally announced bids, offers, and acceptances. Such bid-ask markets are familiar in commodity trading, and indeed scenes of eager traders gathered around a pit in Chicago or a ring in London are a staple of economics textbooks illustrating 'perfectly competitive' markets. Such markets have also been studied experimentally and typically the results strongly confirm the prediction that most of the gains from trade are realized and prices tend to approximate the Walrasian market clearing price, especially if repetition of the market enables traders to learn from experiences; cf. Plott [1982], Smith [1982], and Easley and Ledyard [1983].

In Wilson [forthcoming 1986b] I have attempted to synthesize a construction of sequential equilibria for bid-ask markets from elements that we already know from previous studies of bargaining, auctions, and monopoly. The key idea is to envision the market as a process by which sellers and buyers are endogenously matched into bargaining pairs, and impatience to trade is induced mainly by competitive pressure; in turn, the induced impatience determines the terms of trade, much as in the Dutch auction described above.[14/] Using this approach I establish that at least the necessary conditions associated with the corresponding direct revelation game can be satisfied; the full sufficient conditions for verification of an equilibrium remain to be studied.

The matching and bargaining process works as follows. As in Cramton's bargaining model [1984a, 1984b] using nonstationary strategies, say that an offer is serious if according to the equilibrium it has positive probability of acceptance. For a continuous-time model, one anticipates the existence of a separating equilibrium in pure strategies for which a trader's first serious offer reveals his valuation. Moreover, at any time only the buyer with the highest valuation or the seller with the lowest valuation may find it optimal to make or accept a serious offer. Along the equilibrium path, therefore, the play of the game transpires as follows. The game opens with an initial phase in which no trader makes a serious offer--waiting is the means of signaling that the buyer's valuation is not very high and the seller's valuation is not very low. When a serious offer is made but not accepted, the game enters a second phase in which the revealed trader continues with serious offers (as in a Dutch auction) until one is accepted, thereby revealing the valuation of the acceptor to be the highest if he is a buyer or the lowest if he is a seller. Completion of this second phase with a trade removes the highest-valuation buyer and the lowest-valuation seller from the market; consequently, the ensuing 'subgame' is like the original game except that one fewer buyers and one fewer sellers are present, and the probability distributions of the remaining traders' valuations are truncated by the inference that the buyers remaining have valuations lower than the one last revealed, and the sellers remaining have valuations more than the one last revealed. Of course, if the Dutch auction runs its course with no acceptance, then the game concludes; in this case one can infer that no gains from trade remain. Similarly, if the initial phase produces no serious offer, then no gains from trade are present.[15]

In more detail, the second phase runs as follows. Suppose that the lowest-valuation seller has made a serious offer that has not been accepted. Along the equilibrium path, the buyers are able to infer from this serious offer the seller's valuation, whereas this seller remains uncertain about the buyers' valuations. The continuation game therefore consists of a single-seller, many-buyer bargaining game in which the seller's valuation is presumed known. Generalizing from the one-buyer case with frequent alternating offers, we expect in this case an equilibrium in which only the seller makes serious offers and the buyers wait until the seller's ask price declines sufficiently before one accepts.[16/] Thus effectively the seller is in the position of conducting a Dutch auction against the several remaining buyers who have not yet traded and whose valuations have not been revealed by any serious bid. Intervention by other sellers is precluded by the revelation that the revealed seller's valuation is the lowest and therefore that he can undercut any competitive offer. As the seller continuously reduces his offer towards his (revealed) valuation, each buyer follows a Dutch auction strategy specifying how low the seller's offer must get before accepting.[17/] Of course this acceptance level depends on the buyer's valuation, on the sum of the discount rate and his perceived hazard rate that some other buyer will accept first, and on his expected value of continuation in the ensuing 'subgame' should he fail to trade with the presently revealed seller. Acceleration of the process by the seller is forestalled by the anticipation that this would induce the buyers to reassess a lower estimate of the seller's cost.[18/]

Thus for the seller, his value of continuation when he makes a serious offer is his expected discounted profit from the ensuing Dutch auction with offers descending to his valuation. Similarly, for a buyer contemplating making a first revealing bid, his value of continuation is his expected discounted profit from the ensuing Dutch auction with bids ascending to his valuation.

With this construction of the consequences of making a revealing bid or offer, we can address the determination of the traders' strategies in the initial phase. Consider the case of a seller. At any time the equilibrium predicts how low an ask price must be to have a chance that some buyer will accept; this required level of serious ask prices is increasing over time, since it is associated with the inferred valuation of a seller making such a serious offer. At any instant a seller's decision whether to wait or to make the maximum serious offer trades off two considerations, each the sum of several terms. First, by waiting he obtains a chance that some buyer will enter a serious bid, which is advantageious since his expected profit is greater if he is on the receiving end of the ensuing Dutch auction. Moreover, by waiting and offering a higher serious ask, his profit is increased if it is accepted. And further, waiting induces in the buyers the inference that his valuation is higher, and this inference increases their acceptance prices in the ensuing Dutch auction. Second, however, he incurs two costs. One is the hazard that another seller will intervene with a serious offer and conclude an advantageous trade with the highest-valuation buyer, who will then be unavailable in the ensuing 'sub-game'. The second is the foregone interest on his expected profit were he to conclude a trade by making a serious offer immediately. Both of these costs might in principle be negligible were it not for a particular feature of the equilibrium: there is a positive probability that the initial serious offer is accepted outright. That is, a 'lump' of buyer types find it advantageous to accept the first serious ask price. This is because they have been waiting partly in hopes that some seller will make the first serious offer; when a seller does make a first serious offer, this incentive is removed and they accept.

I am quick to acknowledge that this construction has deficiencies, although I tend to view them less as refutations and more as challenging topics for research. First, it is abundantly clear that the whole approach needs to be substantiated via a corresponding discrete-time model. Second, the off-the-equilibrium-path beliefs that sustain a sequential equilibrium of this type are less than fully convincing. In the Dutch auction, ask prices that are higher than buyers expected are ignored, but the lower ones are interpreted as convincing evidence that the seller's valuation is lower than originally estimated from the first serious offer; as a consequence the buyer's acceptance strategies are necessarily nonstationary, since they depend on the history of the seller's offers after the first serious one. This feature is central to the equilibria constructed by Cramton [1984a, 1984b] for his bargaining model. The device is not uniformly successful, moreover, since it does not deter the lowest-cost seller from accelerating the Dutch auction, for this case Cramton requires a special treatment, in effect reverting to stationarity by assuming that thereafter the buyer(s) expects to be able to capture all the gain from trade. In contrast, we know from Fudenberg, Levine, and Tirole [1983] and from Gul, Sonnenschein, and Wilson [1985] for the bargaining problem in the discrete time case, that if the seller's valuation is common knowledge (rather than inferred from a serious offer) then equilibria with stationary strategies for the buyers exist (uniquely if the seller's valuation is low enough), and most crucially, satisfy the Coase conjecture: if the interval between offers is short (e.g., continuous time) then the buyer captures most of the gains from trade. Presently my view on this matter is that resolution of this difficulty would be most satisfactorily achieved by relaxing two features of the model: that delay is the sole signaling mechanism, and that serious offers are perfectly revealing. More on this below, but suffice it to say that an equilibrium with considerable pooling might be possible. In such an equilibrium a serious offer might be made by a clump of buyer or seller types, and the subsequent refinement of this clump into more finely identified intervals of types might proceed piecemeal, with alternation between the refinements of the sellers' and buyers' identified clumps.

The third 'deficiency' has practical ramifications for the conduct and analysis of experiments with bid-ask markets. The hypothesis that there are precise serious offers anticipated at each time and these are fully revealing as to the traders' valuations, is partly derived from the assumption that the traders' probability assessments are common knowledge. Such common knowledge is rarely present in experiments and never in practice; moreover the equilibrium depends on unrealistic computational abilities, and therefore even if present initially such common knowledge erodes with time. This difference between assumptions and reality is reflected, I think in the absence in experimental protocols of any clear evidence that the predicted signaling and Dutch auction phases are occuring. Presently my conclusion is that the equilibrium described above, even if mathematically correct, is simply one of the many possible equilibria, and unfortunately perhaps, one that is implausible as a positive predicted theory. I suggest therefore that we take it mainly as an instructive exercise, and an indication that as more robust models of bargaining are developed they can be used to develop corresponding models of the intricate behavior that occurs in bid-ask markets.

For experimental purposes, the likely prediction of any model based on endogenous bargaining is that the parties will trade in order of their valuations (if no risk aversion intervenes), that in any transaction the imputed gains from trade are measured relative to the continuation values in an ensuing sub-game, and that these gains will be split in proportions reflecting competitive pressures on the two parties. As a practical matter, the continuation values might be estimated (rather than computed from theoretical considerations) from the data obtained from replications of the market, so that the main test of the theory is one of internal consistency rather relying on absolute predictions that depend crucially on the common knowledge structure and the participants' computational abilities.

This brings me back to the theme with which I opened this section. I think that developments in the theory of bargaining, auctions and other finely structured market processes can be viewed also as steps towards synthetic theories of complex markets. The device I have proposed and illustrated is to interpret complex markets as possibly involving a hierarchy of imbedded bargaining and auction games with endogeous processes of signaling and competitive pressure. These supplant the simplistic assumptions of exogenously specified matching processes and impatience parameters used in bargaining models. I see this approach as a way to develop useful positive hypotheses about realistically complex markets from tractable simple models in which the structural features of equilibria can be derived.

It may be also the only way that the key role of delay and impatience in simple models can be reconciled with the key observation that most markets operate quickly. Indeed, if costly delays and impatience not augmented by competitive pressure, were crucial to the operation of markets then the delay costs would render such markets inefficient quite apart from the realized gains from trade. Alternatively, if trading is rapid and strategies are stationary then informed traders have no incentive to make revealing offers, or when they do they forego most of the gains from trade. What we see in practice is that gains from trade are identified and realized quickly and efficiently, and apparently gains are distributed fairly evenly. My hypothesis is that this success reflects the power of competitive pressure to greatly magnify traders' impatience and thereby to realize the signaling and price determination functions to be performed without significant real delay costs.

Central to this thesis, of course, is the belief that identification of the possibility of gains from trade is crucially important. Were gains from trade known to be present, as for example might be the case in labor negotiations, most models (Rubinstein [1982]; Gul, Sonnenschein, Wilson [1985]; etc.) predict that simply accelerating the rate of offers achieves a quick and efficient determination of the terms of trade--although possibly to the disadvantage of a trader with a commonly known valuation, as in the case of the Coase conjecture. For this reason, current work on bargaining models with incomplete information on both sides has special importance; in contrast to the genre of models that assume foreknowledge of gains from trade and focus exclusively on the terms of trade, these models can reveal more about the possible equilibria that include revelation of the existence of gains from trade.

## 4. Conclusion

I have discussed three main topics: incentive efficiency of trading rules, equilibria of dynamic trading games, and assembling models of complex markets from simpler ingredients. These topics are linked by the theme that it is worthwhile to study the trading rules found so prominently in practice, a theme that happily is shared by those using experimental methods. The familiar trading rules are likely efficient, both individually and as endogenous components of larger assemblies. I think too that we can learn from studying the structural features of the equilibria of models based on the familiar trading rules. The fine details of equilibria evidence the constraints imposed by logical consistency in reconciling the optimal strategies of several interacting agents. These details may be reflections of practical aspects of behavior in actual markets, and moreover they provide hypotheses that are specific enough to be testable. Certainly this has been true in the case of auctions--witness the role of the winner's curse--and I think that elementary bargaining models are already contributing a better understanding of negotiations. Game-theoretic formulations have also been helpful in revealing the deficiencies in previous theories that ignored some of the aspects of strategic behavior; industrial organization has been the main turf for this encounter. Ultimately I would like to see a reconstruction of economic theory to take full account of strategic behavior in dynamic situations with incomplete information. In this endeavor, the exploration of the properties of trading rules is one step in a broader program to construct a convincing theory of price formation.

For the audience of non-economists beyond this Congress, the study of trading rules and other realistic aspects of the micro-structure of markets is likely welcome. Bargaining, auctions, bid-ask markets, and brokered and specialist markets are important economic institutions. Laymen expect economists to say something interesting about these institutions, something more useful than that demand equals supply. When we attain an explanatory theory that encompasses these institutionalized markets, elucidates oligopolistic behavior, and explains the many forms of discriminatory pricing, then economics will have better tools to be a practical science at the micro level.

# APPENDIX

This appendix amplifies several of the models mentioned in the text by presenting explicit, although very brief, formulations. For complete expositions, see the references mentioned.

## A.1. Conditions for Incentive Efficiency of Double Auctions

A criterion sufficient to establish the incentive efficiency of a trading rule is that there exist welfare weights $\alpha_i(v_i)$ for each trader $i$ in the event his valuation is $v_i$ such that no other trading rule achieves a higher value of the welfare measure

$$w \equiv E\ \{\sum_i \alpha_i(v_i)U_i(v_i)\}\ ,$$

where for some selection of the equilibrium $U_i(v_i)$ is trader $i$'s expected gain from trade if his valuation is $v_i$, and $E$ indicates the expectation over the traders' valuations. From the analysis of the associated direct revelation game, Myerson and Satterthwaite [1983] have shown that such a measure can be written as

$$w = E\ \{\sum_{i \in T} k_i u_i(v_i)\} + \text{constants}\ ,$$

where the indicated constants depend only the participation constraint. $T$ is the (random) set of traders who trade, and each $k_i$ is plus or minus one as the trader is a buyer or seller, so feasibility requires that $\sum_{i \in T} k_i = 0$. In the formula, $u_i(v_i)$ is what Myerson calls trader i's <u>virtual</u> valuation. For example, if the sellers' valuations are distributed i.i.d. according to the distribution function $F$ then

$$u_i(v_i) = v_i + \bar{\alpha}_i(v_i)F(v_i)/F'(v_i) ,$$

where

$$\bar{\alpha}_i(v_i) \equiv E\,\{\alpha_i(\tilde{v}_i) \mid \tilde{v}_i < v_i\} .$$

In a double auction, by comparison, the set $T$ of traders whose trade is selected to maximize

$$\sum_{i \in T} k_i \sigma_i(v_i) ,$$

where $\sigma_i$ is the strategy that specifies trader $i$'s submitted bid or ask price depending on his valuation $v_i$. Since this maximization depends only on ordinal comparisons, a double auction is therefore incentive efficient if there is some increasing function $\phi$ such that $\phi\,(\sigma_i(v_i)) = u_i(v_i)$. One function is constructed in Wilson [1985c] that has the required properties if the numbers of buyers and sellers are sufficiently large.

A.2. Characterization of the Monopoly Problem

Consider a seller with zero unit cost and a population of buyers with types in the interval $[0,1]$ such that a buyer of type $x$ has the valuation $f(x)$, where $f$ is increasing. The seller and all buyers use the discount factor $\delta$. If the seller expects that a buyer of type $x$ uses the stationary strategy that accepts any price not exceeding $P(x)$, then along the equilibrium path his value of continuation $V(x)$ when the set of buyers remaining is the interval $[0,x]$ must satisfy the dynamic programing relation:

$$V(x) = \max_{y<x} P(y)[x - y] + \delta V(y) ,$$

and if $y(x)$ is the (maximal) optimal choice here then his optimal price is $p(x) = P(y(x))$ . For the buyer of type $x$ , on the other hand, it is optimal to accept a price $p$ only if waiting for a lower price is not preferable. Along the equilibrium path this reduces to the condition that:

$$f(x) - P(x) = \delta[f(x) - p(x)] .$$

Characterizations of the equilibria determined by these conditions are obtained in Fudenberg, Levine, and Tirole [1983] and Gul, Sonnenschein, and Wilson [1985]. As noted there, randomization by the seller may be necessary off the equilibrium path. An elaborate example is in Wilson [1985d] and others are described in Gul, Sonnenschein, and Wilson [1985].

In the case of bargaining, let $x$ indicate the buyer's type, uniformly distributed on $[0,1]$, and then $f(x)$ is the buyer's valuation if his type is $x$ . Let $V^*(x)$ be the seller's expected discounted value of continuation after a history that enables the seller to infer that the buyer's type lies in the restricted interval $[0,x]$ . If only the seller makes offers then the same conditions, with $V(x) = V^*(x)x$ , characterize equilibria with a stationary strategy for the buyer. If both can make offers but the buyer finds it preferable to make only nonserious offers that the seller is sure to reject, then replace $\delta$ by $\delta^2$ since the seller makes offers only every other period.

## A.3. An Equilibrium of a Bargaining Game

Assume that the seller and buyer alternate offers, and as before the seller's cost is zero and the buyer's valuation is $f(x)$ if his type is

$x \in [0,1]$ , where $x$ is uniformly distributed. Consider an equilibrium with a stationary strategy for the buyer with the reservation price $P(x)$ if his type is $x$ . Along the equilibrium path, if the buyer finds it optimal to make a nonserious counteroffer then the seller's continuation value satisfies the dynamic programming relation specified above (using $\delta^2$ ). Otherwise, it satisfies

$$V(x) = \max_{y<x} P(y)[x - y^* + \delta W(y) ,$$

where $W(y)$ is his continuation value after rejection of the offer $P(y)$ when it is the buyer's turn to counteroffer. In turn, this must satisfy

$$W(y) = q(z)[y - z] + \delta V(z) ,$$

if he expects the buyer to make the serious acceptable counteroffer $q(z)$ if his type is in the interval $[z,y]$ . In this case the buyer's reservation price necessarily satisfies

$$f(y) - P(y) = \delta[f(y) - q(z)] .$$

Grossman and Perry [1984] consider the counteroffer $q(z) = \delta f(z)/1 + \delta$, which is the Rubinstein offer by the buyer expecting that any lower price will be countered by the seller with acceptable offer $f(z)/(1 + \delta)$, since the seller can now infer that the buyer's type lies in the interval $[z,y]$ (they establish conditions for this to be optimal for the seller). Moreover, the lowest type $z$ for which this counteroffer is optimal satisfies:

$$f(z) - q(z) = \delta[f(z) - p(z)] ,$$

where $p(z)$ is the seller's optimal offer when he infers that the buyer's type is in the interval $[0,z]$ if the counteroffer $q(z)$ is not forthcoming.

The example in the text uses the assumption that $f(x) = x$ so that the buyer's valuation is uniformly distributed. As shown in Table 1 (page 22), if the discount factor is sufficiently high then the buyer makes no serious counteroffers. Thus, the Coase conjecture applies and the buyer obtains most of the gains from trade if the discount factor is large. This is an instance of the more general theorem of Gul and Sonnenschein [1985], which also addresses the case that the buyer makes counteroffers if his valuation is sufficiently small; also, these authors allow a more general class of equilibria than allowed by Grossman and Perry.

A.4. Bargaining with Offers only by the Informed Trader

Consider for example the simplified version of the game in continuous time in which both parties use the discount rate $r$, the seller's valuation is 0, and the continuous distribution function of the buyer's valuation is $F$ on an interval $[v_*,1]$. Along the equilibrium path, if the buyer's valuation is $v$ then he bids 0 until he makes his first serious offer $p(v)$ at time $t(v)$. The seller immediately accepts because he expects the bid thereafter to increase towards the buyer's revealed valuation at a rate less than the discount rate, which is also optimal for the buyer given the expectation that the seller will accept such bids. The buyer's delay in making a serious offer is sustained by the seller's strategy. Along the equilibrium path the seller rejects bids of 0 and at any time $\hat{t}$ he accepts bids $\hat{p}$ not less than the predicted serious offer level, as specified above: $\hat{p} > p(t^{-1}(\hat{t}))$. Off the equilibrium path, if at $\hat{t}$ the buyer offers $\hat{p} < p(t^{-1}(\hat{t}))$ *then the seller rejects* and continues with a strategy that is optimal for the 'subgame' with the support of the buyer's distribution truncated to some interval $[a,b]$, where $b = t^{-1}(\hat{t})$ by inference from the previous absence of a serious offer. There are, however, many feasible choices for the lower bound $a = a(\hat{p},\hat{t})$ that satisfy the natural constraints that $a(p(v),t(v)) = v$ and that rejection of $\hat{p}$ is optimal for the seller:

$$\hat{p} < \frac{\int_a^{t-1(\hat{t})} p(x)e^{-r[t(x) - \hat{t}]}dF(x)}{F(t^{-1}(\hat{t})) - F(a)}$$

For example, a plausible choice of $a$ is one that satisfies

$$a - \hat{p} = [a - p(a)]e^{-r[t(a)-\hat{t}]} ,$$

so that $[a,b]$ is the interval of buyer types preferring that the seller accepts $\hat{p}$ at $\hat{t}$ rather than waiting to make their equilibrium offers at later times. Cramton uses the simpler convention that $a = b = p^{-1}(\hat{p})$ . Either way, a special treatment is required for the case that $\hat{p} < p(v_*)$ since in this case a low-value buyer is not necessarily deterred by the seller's revised assessment, and such a buyer has an incentive to accelerate the process: Cramton's device is to assume that in this case the seller expects and only accepts prices of $v^*$ thereafter.

The possibility that this variety of disequilibrium assessments by the seller induces a comparable variety of equilibria is clear from the fact that the buyer's anticipation that the seller will not accept prices lower than $p(v)$ *before* $t(v)$ implies for an optimal strategy only the necessary condition that

$$p'(v)/t'(v) = -r[v - p(v)] \text{ .}$$

This condition determines only one of the two functions $p$ and $t$ given the other; it remains to fix the equilibrium by specifying the seller's interpretation of the signaling significance of the buyer's delay. Thus, although delay is an effective signal for the buyer, various equilibria are possible depending on the seller's prediction of the delay that each buyer type will use to signal his valuation.

### A.5. Impatience in an Auction

Consider a Dutch auction in which a bidder with the valuation $v$ obtains the discounted profit $[v - p(t)]e^{-rt}$ if he is the first to accept the ask price $p(t)$ at the time $t$. Suppose the $n + 1$ bidders' valuations are distributed i.i.d. such that $F(\hat{v})$ is the probability that one's valuation is less than $\hat{v}$. If one expects another bidder with the valuation $\hat{v}$ to accept at time $t(\hat{v})$, then he will choose his acceptance time $s$ to maximize his expected discounted profit:

$$[v - p(s)]e^{-rs}F(t^{-1}(s))^n .$$

A symmetric equilibrium requires that the optimal choice is $s = t(v)$. A necessary condition for this optimum is that:

$$|p'(t(v))| = [v - p(t(v))]\{r + nF'(v)/F(v)|t'(v)|\} .$$

As mentioned in the text, the second term in the curly brackets is the hazard rate that another bidder will intervene with an earlier bid. The interest rate $r$ and this hazard rate add to impute the bidder's impatience in delaying receipt of the immediate profit $v - p(t)$ rather than waiting for the price to decline further before accepting. Either a positive interest rate or a positive hazard rate suffices to induce early acceptance by the bidder. If there are many bidders then the hazard rate is the dominant term. This is an instance of the competitive pressure mentioned in the text.

## FOOTNOTES

1/ See especially Myerson and Satterthwaite [1983] and Gresik and Satterthwaite [1983]. For surveys see Myerson [1985] and Wilson [1985b].

2/ That this is a practical concern is evidenced by the Department of Interior's policy for setting reservation prices on oil leases: The calculation is based on the costs of delay until the lease can be reoffered in a subsequent auction

3/ Impatience may also be influenced by agents' risk aversion and nonlinear preferences for gains,both of which I exclude; cf. Roth [1985].

4/ This result depends on the rule of public offers; private offers directed to particular agents can yield non-Walrasian prices; see Wilson [1985d] for an example.

5/ For the reader who prefers a one-line explanation, go directly to the continuous-time limit: if the schedule of prices over time were not flat, then the seller would prefer to accelerate the process by rescaling time so that the clock runs twice as fast.

6/ Also imposed are a monotonicity assumption and a 'no free screening' assumption. Here, stationarity means essentially that the buyer's response to the seller's current offer does not depend on features of the history that do not affect the seller's equilibrium prediction of the set of buyer types who would accept; in turn, this prediction depends only on the seller's current offer and his probability assessment that motivated this offer.

7/ The non-existence result in Grossman and Perry [1984] depends on the presumption that the buyer makes counteroffers no matter how large is the discount factor.

8/ This equilibrium is analogous to the unique subgame-perfect equilibrium for the monopoly problem with a population of buyers having two types; cf. Wilson [1985d]. Bikhchandani's equilibrium has a further desirable property: unlike Rubinstein's it is preserved under the insertion of dummy moves by the seller after the buyer rejects but before the buyer counteroffers. The addition of dummy moves of this sort allows the seller to revise his beliefs about the buyer's type more frequently; in the present example it excludes an equilibrium in pure strategies, provided that (as both Rubinstein and Bikhchandani assume) zero probabilities can never be revised to be positive. Preservation of an equilibrium under irrelevant transformations, such as the addition of dummy moves, is a central requirement for the equilibrium to be stable.

9/ However, as the probability tends to 1 that the buyer is impatient, it becomes sure that the seller and impatient buyer trade immediately at the Rubinstein price were the buyer known to be impatient.

10/ Also interesting is that if the seller's cost exceeds the buyer's valuation then in finite time one or the other can conclude that no gains from trade exist and cease the bargaining.

11/ Cramton's equilibria rely on two special features that future research could usefully investigate. One is the assumption that when both parties' valuations are revealed, trade occurs at the Rubinstein price were their valuations common knowledge and the trading rule one of alternating offers; whereas when only one has revealed, revisions of beliefs remain possible if the revealed trader deviates by attempting to accelerate the process. Thus, 'probability one' events are taken to be equivalent to common knowledge in the one case and not in the other. The second is that deviation by the seller with revealed valuation that is the minimal one possible is deterred by the optimistic belief of the buyer that he can expect minimal prices in the future; thus, at the seller's minimal possible valuation, beliefs are not continuous in the seller's offer. It is unclear whether these assumptions can be weakened while retaining the form of Cramton's equilibria. Perhaps the more detailed study of trading rule (A) in a discrete time model can clarify these issues.

12/ On the other hand, Gale [1985] points out that the Rubinstein-Wolinsky result is 'Walrasian" if market clearing is interpreted in a flow sense: given the equal arrival rates of buyers and sellers (even though the 'stocks' of buyers and sellers are unequal) any price between their two valuations induces departures at a rate equal to the arrival rate; in this interpretation the unequal stocks of buyers and sellers and the matching process are relevant mainly to the determination of which price in the interval of flow-clearing prices is selected. This points out that interpretations of Walrasian models depend on whether market clearing is interpreted in a stock or flow sense.

13/ Part of this assumption is that traders' endowments are sufficiently diverse. The finite stopping time is ensured by requiring the total measure of the agents to be finite; this precludes a stationary matching process.

14/ In Wilson [forthcoming 1986b] the exposition assumes for simplicity that the agents' discount rate is zero, but this leads to an incomplete determination of the strategies in the endgame in which only a single buyer or seller remains (since a single trader on one side of the market has no induced impatience from competitive pressure). Making the discount rate positive removes this indeterminancy, but of course then one requires nonstationary strategies to sustain the equilibrium against deviations in which a trader would prefer to accelerate the process.

15/ In Wilson [forthcoming 1986b] I assume a finite allowed time and a zero interest rate, in which case the initial phase could end with a small positive probability of gains from trade remaining.

16/ That is, an equilibrium with serious counteroffers by the buyers cannot be sustained because such bids reveal the buyer's valuation and are therefore disadvantageous for the buyers compared to the opportunity to wait for further reductions in the seller's ask price.

17/ A serious counteroffer is excluded here because, as we have seen in Section 2, it is advantageous for the buyers to let the seller make all the offers.

18/ As mentioned previously, unfortunately a special treatment is required for the case that the seller's cost is the minimal one; as in Cramton [1984a], in this case invoke the Coase conjecture in the form that buyers expect to obtain all the gains from trade.

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